Non-SUSY Heterotic Strings

via Free Fermions and Orbifolds

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Based on papers:

- *Towards The Classification of Tachyon Free Models From Tachyonic Ten Dimensional Heterotic Strings*, arXiv: 2006.11340 [Faraggi, Percival & VGM]
- *Type 0* $\mathbb{Z}_2 \times \mathbb{Z}_2$ *Heterotic String Orbifolds and Misaligned Supersymmetry*, arXiv: 2010.06637 [Faraggi, Percival & VGM]
- *Classification of Non-Supersymmetric Pati-Salam Heterotic String Models*, arXiv: 2011.04113 [Faraggi, Percival & VGM]
- *Type* $\bar{0}$ *Heterotic String Orbifolds*, arXiv: 2011.12630 [Faraggi, Percival & VGM]
- *Towards Classification of* $\mathcal{N} = 1$ *and* $\mathcal{N} = 0$ *Flipped* $SU(5)$ *Asymmetric* $\mathbb{Z}_2 \times \mathbb{Z}_2$ *Heterotic String Orbifolds)*, arXiv: 2202.04507 [Faraggi, Percival & VGM]

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Introduction

Non-SUSY Strings Approach #1

Most of the effort over the years has gone into the construction of SUSY theories:

SUSY Theory in 10D

$\downarrow$

Compactify to 4D (Orbifold, CY, $\cdots$)

$\downarrow$

SUSY Theory in 4D

$\downarrow$

Break SUSY (SS, Branes, $\cdots$)

Benefits: Good handle on finiteness and other quantities
No Tachyons
...

Non-SUSY Strings Approach #2

There are other methods by which to construct viable Non-SUSY theories in 4D.
An example:

Non-SUSY Theory in 10D (Tacyonic)

$\downarrow$

Compactify to 4D (-ish)

$\downarrow$

Non-SUSY Theory in 4D (Non-Tachyonic)

Benefits: A-priori Non-Supersymmetric
No Tachyons
Many novel models to explore
...

Free Fermions vs Orbifolds

Both can be viewed as Toroidal Orbifold Compactifications:

Free Fermions vs Orbifolds

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Free Fermionic Construction

+ Easy to check for many pheno features
+ Very convenient for large scans of landscape
+ Don't always need geometric picture

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Orbifold Compactification

+ We have a good geometric understanding
+ Moduli depence of quantities is more easily available

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Orbifold Compactification

+ We have a good geometric understanding
+ Moduli depence of quantities is more easily available
- Harder to do large scans of landscape and check for pheno

Why not use both?

Our aim is to use the benefits both formulations in order to analyse the landscape:

Construct 4D theories using Free Fermions

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Reinstate dependence on the moduli

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Stability?

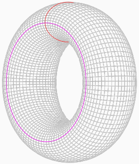
Free Fermionic Construction

Free Fermionic Construction

Instead of a geometric compactification, extra degrees of freedom are taken to be free worldsheet fermions. We hence have 64 (20L,44R) fermions.

To define a model in 4D we need:

- A set of basis vectors b_i



$$\{\psi^\mu, \chi^{1,\cdots,6}, \underbrace{y^{1,\cdots,6}, \cdots \mid \bar{y}^{1,\cdots,6}, \cdots}_{\text{Internal Lattice}}, \underbrace{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,\cdots,3}, \cdots}_{\text{Complex}}\}$$

- A matrix of generalised GGSO phases $C(b_i, b_j)$

$$Z_F = \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_{f}^{64} Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}.$$

Partition Function

Full Partition Function for Free Fermionic models:

$$Z_{Tot} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, Z_B\, Z_F$$

- $\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}$: Integral over the inequivalent tori parametrised by the modular parameter $\tau = \tau_1 + i\tau_2$.

Partition Function

Full Partition Function for Free Fermionic models:

$$Z_{Tot} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \, Z_B \, Z_F$$

- Fermionic Contribution:

$$Z_F = \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}$$

$$Z\begin{bmatrix}a\\ b\end{bmatrix} = \left(\vartheta\begin{bmatrix}a\\ b\end{bmatrix}/\eta\right)^{1/2}$$

Partition Function

Full Partition Function for Free Fermionic models:

$$Z_{Tot} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_B Z_F$$

- Bososnic Contribution:

$$Z_B = \frac{1}{\tau_2} \frac{1}{\eta^2 \bar{\eta}^2}$$

Partition Function and Cosmological Constant

Full Partition Function for Free Fermionic models:

$$Z_{Tot} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} \frac{1}{\eta^2\bar{\eta}^2} \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}$$

This is the one-loop partition function for our theory, i.e. the one-loop vacuum energy $\rightarrow \Lambda$.

We have to evaluate the integral of the form

$$Z_{Tot} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} \frac{1}{\eta^{12}\bar{\eta}^{24}} \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f \vartheta\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}^{1/2}$$

Can be done using an expansion in terms of $q := e^{2\pi i\tau}$, i.e

$$Z = \sum_{n.m} a_{mn} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n.$$

$d\tau_1 \longrightarrow$ analytic
$d\tau_2 \longrightarrow$ numeric

q-Expansion of Z

$$\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n = \begin{cases} \infty & \text{if} \quad m+n<0 \ \wedge \ m-n \notin \mathbb{Z}\backslash\{0\} \\ \text{Finite} & \text{Otherwise.} \end{cases}$$

- On-Shell Tachyons cause divergence
- Off-Shell Tachyons allowed (necessary)

$$\text{Modular invariance} \longrightarrow m-n \in \mathbb{Z}.$$

q-Expansion of Z

Allowed states:

$$a_{mn} = \begin{pmatrix}
0 & 0 & a_{-\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{-\frac{1}{2}\frac{1}{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & a_{-\frac{1}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{-\frac{1}{4}\frac{3}{4}} & 0 & 0 \\
a_{0-1} & 0 & 0 & 0 & a_{00} & 0 & 0 & 0 & a_{01} & 0 \\
0 & a_{\frac{1}{4}-\frac{3}{4}} & 0 & 0 & 0 & a_{\frac{1}{4}\frac{1}{4}} & 0 & 0 & 0 & \ddots \\
0 & 0 & a_{\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{\frac{1}{2}\frac{1}{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & a_{\frac{3}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{\frac{3}{4}\frac{3}{4}} & 0 & 0 \\
a_{1-1} & 0 & 0 & 0 & a_{10} & 0 & 0 & 0 & a_{11} & 0 \\
0 & \ddots & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & \ddots
\end{pmatrix}$$

Coefficients $a_{mn} = N_b - N_f$ at specific mass level.
For SUSY Theories $a_{mn} = 0 \; \forall m, n$

Some Results form Classification

SO(10) and Pati-Salam Models

We have explored SO(10) and Pati-Salam models with the structure:

Non-SUSY Tacyonic 10D Theory

$\downarrow$

Compactify to 4D using Free Fermions

$\downarrow$

Non-SUSY Theory in 4D with Tachyons Projected

$SO(10)$: arXiv:2006.11340 [Faraggi, Percival & VGM]
Pati-Salam: arXiv:2011.04113 [Faraggi, Percival & VGM]

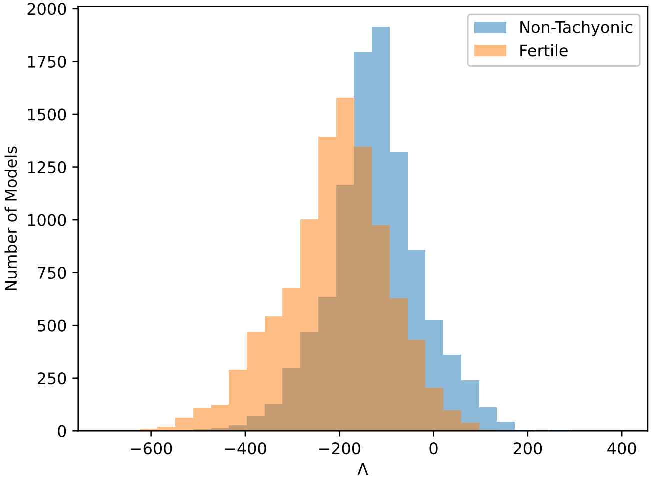
Some interesting results...

Classification of Phenomenological Features

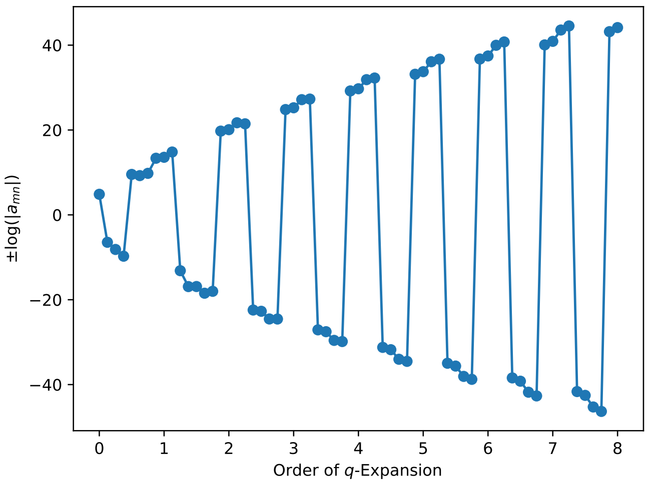
	Constraints	Total models in sample	Probability
	No Constraints	2×10^9	1
(1)	+ Tachyon-Free	10741667	5.37×10^{-3}
(2)	+ No Observable Enhancements	10741667	5.37×10^{-3}
(3)	+ No Hidden Enhancements	9921843	4.96×10^{-3}
(4)	+ $N_{16} - N_{\overline{16}} \geq 6$	69209	3.46×10^{-5}
(5)	+ $N_{10} \geq 1$	69013	3.45×10^{-5}
(6)	+ $a_{00} = N_b^0 - N_f^0 = 0$	3304	1.65×10^{-6}

Phenomenological statistics from sample of 2×10^9 $SO(10)$ *models.*

Distribution of Cosmological Constant



Misaligned SUSY



Corners of the Landscape

What other interesting models can one get from a non-SUSY tachyonic 10D starting point?

Non-SUSY Tacyonic 10D Theory

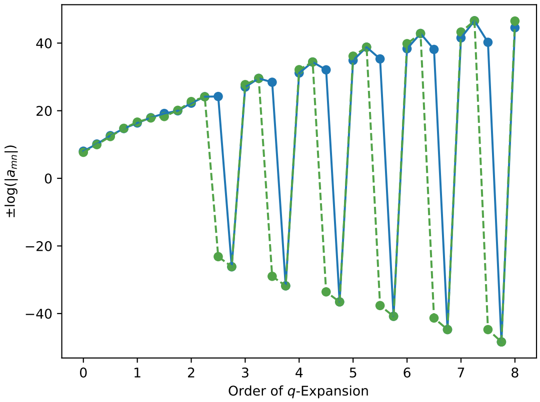
↓

Compactify to 4D using Free Fermions

No Massless Fermions	No Twisted Massless Bosons
(Cannot project tachyons)	(Non-tachyonic)
arXiv:2010.06637	arXiv:2011.12630
[Faraggi, Percival & VGM]	[Faraggi, Percival & VGM]

Misaligned SUSY - Interesting Observation

We observe the Boson-Fermion oscillation of Misaligned SUSY even for tachyonic models. arXiv:2010.06637 [Faraggi, Percival & VGM]



Misaligned SUSY - Origins

$$\text{Finiteness} \quad \xleftrightarrow{?} \quad \text{Misaligned SUSY}$$

Misaligned SUSY - Origins

[Carlo Angelantonj's Talk]

$$\downarrow$$

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↓

Finiteness $\overset{?}{\longleftrightarrow}$ Misaligned SUSY

↑ ↗

Modular Invariance

Misaligned SUSY - Origins

[Carlo Angelantonj's Talk]

↓

Finiteness $\overset{?}{\longleftrightarrow}$ Misaligned SUSY

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Modular Invariance

Misaligned SUSY - Origins

[Carlo Angelantonj's Talk]

$\downarrow$

Finiteness $\overset{?}{\longleftrightarrow}$ Misaligned SUSY

$\uparrow$ $\nearrow$ $\leftarrow$ [Flavio Tonioni's Talk]

Modular Invariance

Misaligned SUSY — arXiv: 9402.006, 9409.114, 9503.055 [Dienes et.al.]

From M.I. — arXiv: 1012.5091 [Angelantonj et.al.]

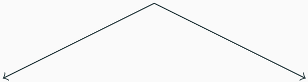
In Open Strings — arXiv: 2110.11973, 2012.04677 [Cribiori et.al.]

Asymmetric Orbifolds

Construction Issues

There are possible issues with these types of constructions:

No handle on geometric moduli



Λ may not be at minimum

Tachyon projections not protected

Plans for Asymmetric Orbifolds

See e.g. [Stefan Groot Nibbelink's Talk]

Combine Two Methods

Assymetric Shifts | arXiv: 2202.04507 [Faraggi, Percival & VGM]

and

Orbifold Techniques | arXiv: 1608.04582 [Florakis & Rizos]
| arXiv: 1502.03087 [Abel et.al.]

Plans for Asymmetric Orbifolds

Construct 4D theories using Free Fermions corresponding to Asymmetric Orbifolds

$\downarrow$

Choose asymmetric shifts such that some moduli are projected

$\downarrow$

Select for wanted pheno features

$\downarrow$

Translate to the corresponding orbifold model

$\downarrow$

Reinstate dependence on the moduli and calculate potential in the unfixed directions

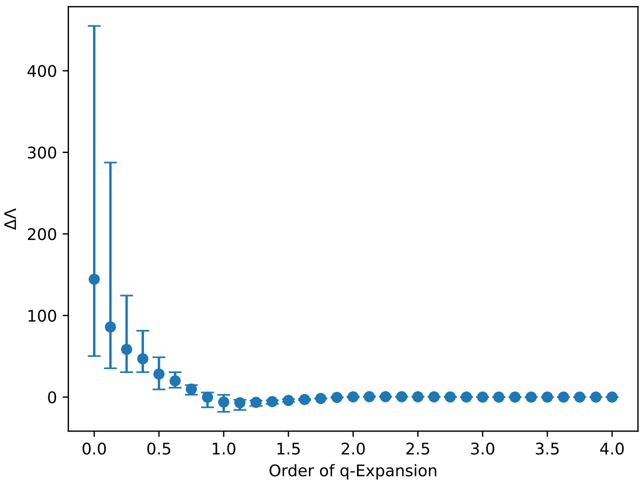
Conclusion

Conclusion

- Free Fermionic Construction handy to get phenomenology.
- Much to explore for Non-SUSY Theories.
- Even Tachyonic Non-SUSY 10D Theories can lead to viable 4D vacua.
- Asymmetric orbifolds are interesting to study stability.
- Both free fermionic and orbifold methods can be used simultaneously.

Thank You!

Convergence of Cosmological Constant



Write partition function of a model, e.g. $\{\mathbf{1}, S, b_1, b_2\}$, in a form that emphasizes the internal structure

$$
\begin{aligned}
Z = & \frac{1}{\eta^{12}\bar{\eta}^{24}} \sum_{a,b} e^{i\pi(a+b+\mu ab)} \sum_{h_1,h_2,g_1,g_2} \sum_{\sigma,\rho} e^{i\pi(\Psi+\Phi)} \\
& \times \vartheta[^a_b]\, \vartheta[^{a+h_2}_{b+g_2}]\, \vartheta[^{a+h_1}_{b+g_1}]\, \vartheta[^{a-h_1-h_2}_{b-g_1-g_2}] \\
& \times \Gamma_{6,6}\begin{bmatrix}\sigma,h_1,h_2\\ \rho,g_1,g_2\end{bmatrix} \\
& \times \bar{\vartheta}[^{\sigma+h_2}_{\rho+g_2}]\bar{\vartheta}[^{\sigma+h_1}_{\rho+h_2}]\, \bar{\vartheta}[^{\sigma-h_1-h_2}_{\rho-g_1-g_2}]^5\, \bar{\vartheta}[^\sigma_\rho]^9.
\end{aligned}
$$

Internal compactification lattice can be isolated as

$$
\Gamma_{6,6}\begin{bmatrix}\sigma,h_1,h_2\\ \rho,g_1,g_2\end{bmatrix} = \left|\vartheta[^\sigma_\rho]\vartheta[^{\sigma+h_2}_{\rho+g_2}]\right|^2 \left|\vartheta[^\sigma_\rho]\vartheta[^{\sigma+h_1}_{\rho+g_1}]\right|^2 \left|\vartheta[^\sigma_\rho]\vartheta[^{\sigma-h_1-h_2}_{\rho-g_1-g_2}]\right|^2.
$$

We can see that this corresponds to an orbifold model with point group $\mathbb{Z}_2 \times \mathbb{Z}_2$:

$$h_1 \longrightarrow \mathbb{Z}_2^{(1)} \textit{ twist}$$

$$h_2 \longrightarrow \mathbb{Z}_2^{(2)} \textit{ twist}$$

$$h_1 + h_2 \longrightarrow \mathbb{Z}_2^{(1)} \ \& \ \mathbb{Z}_2^{(2)} \textit{ twist}$$

Having developed a picture of the corresponding orbifold, we can reinstate dependence on moduli

$$\Gamma_{6,6} \begin{bmatrix} \sigma, h_1, h_2 \\ \rho, g_1, g_2 \end{bmatrix} \longrightarrow \Gamma_{6,6} \begin{bmatrix} \sigma, h_1, h_2 \\ \rho, g_1, g_2 \end{bmatrix} (T, U),$$

such that

$$\Gamma_{6,6} \begin{bmatrix} \sigma, h_1, h_2 \\ \rho, g_1, g_2 \end{bmatrix} (T = i, U = (1+i)/2) = \Gamma_{6,6} \begin{bmatrix} \sigma, h_1, h_2 \\ \rho, g_1, g_2 \end{bmatrix}.$$